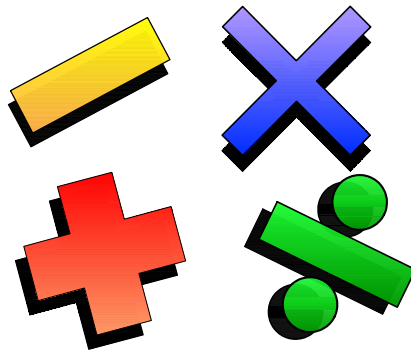




GCSE

MATHEMATICS

The Rules of Algebra



Simplifying Algebraic Expressions

Like Terms

$$a + a = 2a$$

$$6a - 2a = 4a$$

$$5a - 5a = 0$$

Collecting Like Terms

$$2a + 3b + 4a + b = 6a + 4b$$

$$6a + 5b - 2a + 2b = 4a + 7b$$

Multiplication & Division

$$a \times a = a^2$$

$$2 \times a \times b = 2ab$$

$$a \times a \times a = a^3$$

$$a \div b = \frac{a}{b}$$

$$a \div a = 1$$

Rules of Indices

$$a^3 \times a^2 = a^5$$

$$a^6 \div a^4 = a^2$$

$$(a^3)^4 = a^{12}$$

The above satisfy the following general rules:

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

Two further rules for indices are:

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

Indices involving fractions:

$$a^{\frac{1}{n}} = {}^n\sqrt{a}$$

$$a^{\frac{m}{n}} = ({}^n\sqrt{a})^m = {}^n\sqrt{(a^m)}$$

Further Expressions

$$a^2 + a^2 = 2a^2$$

$$4a^2 \times 2a^3 = 8a^5$$

$$5a^3 \div 3a = \frac{5a^3}{3a} = \frac{5}{3}a^2$$

$$\sqrt{9a^2} = 3a$$

Rules Involving Negative Quantities

$$a + (+b) = a + b$$

$$a - (+b) = a - b$$

$$a + (-b) = a - b$$

$$a - (-b) = a + b$$

$$(+a) \times (+b) = ab$$

$$(+a) \times (-b) = -ab$$

$$(-a) \times (+b) = -ab$$

$$(-a) \times (-b) = ab$$

$$(+a) \div (+b) = \frac{a}{b}$$

$$(+a) \div (-b) = -\frac{a}{b}$$

$$(-a) \div (+b) = -\frac{a}{b}$$

$$(-a) \div (-b) = \frac{a}{b}$$

Rules for Removing Brackets

$$a + (b + c) = a + b + c$$

$$a + (b - c) = a + b - c$$

$$a - (b + c) = a - b - c$$

$$a - (b - c) = a - b + c$$

Examples:

$$4(a + 2b) = 4a + 8b$$

$$a(3a - 2b) = 3a^2 - 2ab$$

$$2a(3a + 4b - 2) = 6a^2 + 8ab - 4a$$

$$3(a + 3b) + 5(2a - b) = 3a + 9b + 10a - 5b = 13a - 4b$$

$$4(a + 2b) + 3(a - 4b) = 4a + 8b + 3a - 12b = 7a - 4b$$

$$3(a + b) - (2a + 6b) = 3a + 3b - 2a - 6b = a - 3b$$

$$5(2a + b) - 3(3a - b) = 10a + 5b - 9a + 3b = a + 8b$$

Equations

$$a + 12 = 19$$

Subtract 12 from both sides

$$a = 7$$

$$a - 6 = 14$$

Add 6 to both sides

$$a = 20$$

$$4a = 16$$

Divide both sides by 4

$$a = 4$$

$$\frac{a}{6} = 3$$

Multiply both sides by 6

$$a = 18$$

The golden rule for solving equations is ‘change the side, change the sign’.

Example:

$$12a - 10 = 5a + 11$$

Subtract 5a from both sides

$$7a - 10 = 11$$

Add 10 to both sides

$$7a = 21$$

Divide both sides by 7

$$a = 3$$

Algebraic Fractions

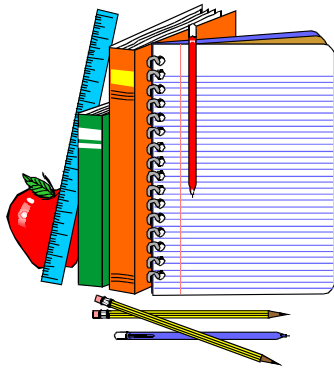
$$\frac{5a^3}{15a} = \frac{a^2}{5}$$

$$\frac{a}{5} + \frac{3a}{4} = \frac{4a + 15a}{20} = \frac{19a}{20}$$

$$\frac{2a}{3} - \frac{a}{4} = \frac{8a - 3a}{12} = \frac{5a}{12}$$

$$\frac{2a}{3b} \times \frac{6b^2}{a} = \frac{12ab^2}{3ab} = 4b$$

$$\frac{3a^3}{b^2} \div \frac{a}{b} = \frac{3a^3}{b^2} \times \frac{b}{a} = \frac{3a^2}{b}$$



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